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ON THE AXIOMATIC THEORY OF MULTISTATE COHERENT STRUCTURES.(U)

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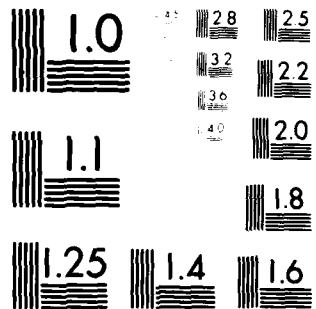
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ON THE AXIOMATIC THEORY OF
MULTISTATE COHERENT STRUCTURES

by

Wagner de Souza Borges^{*†}

and

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ON THE AXIOMATIC THEORY OF MULTISTATE
COHERENT STRUCTURES

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1. ABSTRACT

Mathematical models for multistate reliability systems of multistate components have been proposed by Barlow & Wu (1978), El Neweihi et al (1978) and Griffiths (1981). Unlike the approach used by Barlow & Wu, the other authors preferred to establish their classes of models through sets of axioms, all extending the early binary notions and all containing as special cases the class of models suggested by Barlow & Wu. Since the Barlow & Wu approach is essentially set theoretic, and since in the other two approaches these models were not characterized among the larger classes, one question that arises is whether these models can be characterized by a set of axioms in the same way as their counterparts. In this paper we do just that and obtain a better understanding of Barlow & Wu models.

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2. INTRODUCTION, NOTATION AND TERMINOLOGY

A central problem in reliability theory is to determine the relationship between the reliability of a complex system and the reliabilities of its components. When the system and its components are considered to be in either the functioning (state 1) or the fail (state 0) state, the theory of binary coherent structures, as developed in Barlow & Proschan (1975), provides an adequate model that serves as a unifying foundation for mathematical and statistical aspects of reliability theory.

However, in many practical situations, systems, as well as their components, can assume a wide range of performance levels, thus motivating the development of mathematical models which generalize the binary. Models, representing multistate systems of multistate components, have been investigated by Barlow & Wu (1978), El-Newehi et al (1978) and Griffiths (1981). While the approach used by Barlow & Wu is based on the minimal path and minimal cut representations of the binary coherent structure functions (See Barlow & Proschan, 1975, p. 9), an axiomatic approach is used by El-Newehi et al and Griffiths to develop their models. It turns out that the later developments include as a special proper subclass of multistate system structures, the one introduced earlier by Barlow & Wu.

In this paper, we establish verifiable conditions that characterize the Barlow & Wu models, within a larger class of multistate system structures which contains all the models introduced by El-Newehi et al.

Let $S = \{0, 1, \dots, m\}$ denote the set of all possible states of both the system and the components and let $C = \{1, 2, \dots, n\}$ be the component set. The vector $\underline{x} = (x_1, \dots, x_n)$, in S^n , denotes the vector of states of each component and we write $\underline{x} \leq \underline{z}$ whenever $x_i \leq z_i$, for $i = 1, 2, \dots, n$. If $\underline{x} \leq \underline{z}$ and $x_i < z_i$ for some i , we write $\underline{x} < \underline{z}$. Other special notation include:

$$(k_i, \underline{x}) \equiv (x_1, \dots, x_{i-1}, k, x_{i+1}, \dots, x_n), \quad k \in S, i \in C$$

$$\underline{k} = (k, k, \dots, k), \quad k \in S.$$

The class of multistate system structures introduced by El-Newehi et al is the following:

DEFINITION 2.1. - A function $\xi: S^n \rightarrow S$, is called an EPS multi-state system structure if:

- (2.1) ξ is monotone non-decreasing;
- (2.2) $\xi(\underline{k}) = k$, $k \in S$;
- (2.3) for each $i \in C$ and $k \in S$, there exists $\underline{x} \in S^n$, such that $\xi(k_i, \underline{x}) = k$ and $\xi(\ell_i, \underline{x}) \neq k$ if $\ell \neq k$.

Let now $P = \{P_i: 1 \leq i \leq p\}$ be a Sperner covering of C , i.e., a collection of non-empty subsets of C such that $P_i \not\supset P_j$ whenever $i \neq j$ and $\bigcup_i P_i = C$ (When the covering condition is dropped we shall refer to P simply as a Sperner system on C). It is easy to see that

$$\xi(\underline{x}) = \max_{1 \leq j \leq p} \min_{i \in P_j} x_i, \quad \underline{x} \in S^n,$$

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defines an EPS multistate system structure, and models of this form were first suggested by Barlow & Wu. Multistate system structures of this form will be referred here as BW. Notice that the class \mathcal{P} of subsets of C clearly satisfy the requirements for being the min path sets of a binary coherent structure.

A concept analog to that of a min-path vector for an EPS multistate system structures has been introduced by El-Newehi et al which can be used, as in the binary case, to determine system state.

DEFINITION 2.2 - A vector $\underline{x} \in S^n$ is called a connection vector to level k if $\phi(\underline{x}) = k$. Furthermore, if $\phi(\underline{z}) < k$, whenever $\underline{z} < \underline{x}$, we say that \underline{x} is critical. ///

For $k = 1, 2, \dots, m$ let C_k denote the set of all critical connection vectors to level k , and for $\underline{x} \in C_k$ put

$$C_k(\underline{x}) = \{i: x_i \geq k\}.$$

Then, it is shown in El-Newehi et al (1978) that for $k = 1, 2, \dots, m$, we have $\bigcup_{\underline{x} \in C_k} C_k(\underline{x}) = C$ and that $\xi(\underline{x}) \geq k$ if and only if $\underline{x} \geq \underline{z}$ for some $\underline{z} \in C_\ell$ and some $\ell \geq k$.

3. CHARACTERIZATION OF BW MULTISTATE SYSTEM STRUCTURES

From the reliability point of view, the class of natural model candidates for multistate system structures consists of the functions $\xi: S^n \rightarrow S$ such that

(3.1) ξ is monotone non-decreasing

and

(3.2) $\xi(\underline{0}) = m - \xi(\underline{m}) = 0$,

and which from now on will bear the name of multistate system structures (MSS). The class of all MSS's will be denoted by M .

Note that conditions (3.1) and (3.2) respectively state that system does not degrade when one or more components upgrade, and that whenever all the components fail, or work at best performance levels, the system either fail, or work at its best performance level, respectively.

For MSS's, the notion of critical connection vector can be easily extended to provide a corresponding version of the result mentioned on the last paragraph of section 2.

DEFINITION 3.1 - Let $\xi: S^n \rightarrow S$ be an MSS. For $k = 1, 2, \dots, m$ we say that $\underline{x} \in S^n$ is an upper k -vector if $\xi(\underline{x}) \geq k$. Furthermore, an upper k -vector is called critical if $\xi(\underline{z}) < k$ whenever $\underline{z} < \underline{x}$. The set of all critical upper k -vectors will be denoted by P_k and for $\underline{x} \in P_k$ we shall let

$$P_k(\underline{x}) = \{i: x_i > 0\}. \quad ///$$

THEOREM 3.2 - Let $\xi: S^n \rightarrow S$ be an MSS. Then, for $k = 1, 2, \dots, m$, $\xi(\underline{x}) \geq k$ iff $\underline{x} \geq \underline{z}$ for some $\underline{z} \in P_k$.

PROOF - Sufficiency is obvious. To prove necessity consider the procedure of successively decrease the values of each component of \underline{x} , subject to the restriction that the value of ξ does not drop below k . Since $\xi(\underline{0}) = 0$, this procedure stops when we reach a vector $\underline{z} \in S^n$ for which $\xi(\underline{z}) \geq k$ and $\xi(\underline{w}) < k$ whenever $\underline{w} < \underline{z}$. Clearly $\underline{x} \geq \underline{z}$ and $\underline{z} \in P_k$. ///

The class M contains as a special subclass, the EPS, and hence be BW, multistate system structures defined in Section 2. What we shall do next is study the behavior of elements of M under property P stated below, and show that this property captures the axiomatic essence of the BW multistate system structures when imposed on the suitable subclass of M .

(3.3) PROPERTY P : If $k = 1, 2, \dots, m$ and $\xi(\underline{x}) \geq k$, there exists $\underline{z} \in \{0, k\}^n$ such that $\underline{z} \leq \underline{x}$ and $\xi(\underline{z}) \geq k$.

LEMMA 3.3 - Let $\xi: S^n \rightarrow S$ be an MSS satisfying property P . Then

$$\xi(\{0, k\}^n) = \{0, k\}, \quad k = 1, 2, \dots, m$$

iff $\xi(\{0, m\}^n) = \{0, m\}$.

PROOF - The "only if" part is automatic. To prove the "if" statement let us first notice that under property P , we must have $\xi(\underline{x}) \leq k$, for all $\underline{x} \in \{0, k\}^n$ and each $k = 1, 2, \dots, m$.

Since the assertion is now valid for $k = m$, assuming that

it holds for some k , $1 < k < m$, and letting $\underline{x} \in \{0, k-1\}^n$, we have that $\underline{y} = \frac{k}{(k-1)} \underline{x} \in \{0, k\}^n$ so that $\xi(\underline{x}) = 0$ if $\xi(\underline{y}) = 0$. If on the other hand $\xi(\underline{y}) = k$, it follows from property P that there exists $\underline{z} \in \{0, k-1\}^n$ such that $\underline{z} \leq \underline{y}$ and $\xi(\underline{z}) \geq k-1$, and since $\underline{z} \leq \underline{x} \leq \underline{y}$, we must have $\xi(\underline{x}) \geq k-1$. The observation made in the beginning of the proof shows that in fact $\xi(\underline{x}) = k-1$ and the proof is complete. ///

LEMMA 3.4 - Let $\xi: S^n \rightarrow S$ be an MSS satisfying property P and assume that $\xi(\{0, m\}^n) = \{0, m\}$. For $k = 1, 2, \dots, m$, we have $\underline{x} \in P_k$ iff $\frac{1}{k} \underline{x} \in P_1$.

PROOF - Let $\underline{x} \in P_k$. Since property P holds we have that $\underline{x} \in \{0, k\}^n$, so that $\frac{1}{k} \underline{x} \in \{0, 1\}^n$, and from property P there exist $\underline{z} \in \{0, 1\}^n$ such that $\underline{z} \leq \frac{1}{k} \underline{x}$ and $\xi(\underline{z}) \geq 1$. Obviously $\underline{z} \leq \frac{1}{k} \underline{x}$, so that $\xi(\frac{1}{k} \underline{x}) \geq 1$. To show that $\frac{1}{k} \underline{x} \in P_1$ notice that if $\xi(\underline{w}) \geq 1$ for some $\underline{w} < \frac{1}{k} \underline{x}$, there must exist $\underline{y} \in \{0, 1\}^n$ such that $\underline{y} \leq \underline{w}$ and $\xi(\underline{y}) \geq 1$. Then, $k\underline{y} \in \{0, k\}^n$, $\xi(k\underline{y}) \geq \xi(\underline{y}) \geq 1$ and from Lemma 3.3 we must have $\xi(k\underline{y}) = k$ and $k\underline{y} < \underline{x}$, contradicting the fact that $\underline{x} \in P_k$.

The converse is proven using the same type of arguments. ///

As a consequence of the results of Lemmas 3.3 and 3.4 we have the following general result regarding MSS's.

THEOREM 3.5 - Let $\xi: S^n \rightarrow S$ be an MSS. Then

$$(3.4) \quad \xi(\underline{x}) = \max_{1 \leq j \leq p} \min_{i \in P_j} x_i$$

for all $\underline{x} \in S^n$, where $\{P_j: 1 \leq j \leq p\}$ is a Sperner system on

$C = \{1, 2, \dots, n\}$, iff $\xi(\{0, m\}^n) = \{0, m\}$ and property P is satisfied.

PROOF - It is easy to see that if $\xi: S^n \rightarrow S$ is of the form (3.4) then $\xi(\{0, m\}^n) = \{0, m\}$ and property P holds.

If, on the other hand, these two conditions are satisfied, we have

$$\begin{aligned} \xi(\underline{x}) \geq k & \text{ iff } \underline{x} \geq \underline{z} \text{ for some } \underline{z} \in P_k & (\text{theorem 3.2}) \\ & \text{ iff } \underline{x} \geq k\underline{w} \text{ for some } \underline{w} \in P_1 & (\text{lemma 3.4}) \\ & \text{ iff } \min_{i \in P_1(\underline{w})} x_i \geq k \text{ for some } \underline{w} \in P_1 \\ & \text{ iff } \max_{\underline{w} \in P_1} \min_{i \in P_1(\underline{w})} x_i \geq k. \end{aligned}$$

The result now follows if we observe that $\{P_1(\underline{w}); \underline{w} \in P_1\}$ is a Sperner system of subsets of C. ///

We remark at this point that the class of functions $\xi: S^n \rightarrow S$ satisfying conditions (3.1), (3.3) and

$$(3.5) \quad \xi(\{0, m\}^n) = \{0, m\}$$

is still larger than the class of BW multistate system structures, and the reason being is that the Sperner system $\{P_j: 1 \leq j \leq p\}$ in the representation (3.4) of ξ may not cover C.

Nevertheless this covering property can be achieved through the following notion of component relevance:

$$(3.6) \quad \text{for each } i \in C = \{1, 2, \dots, n\}, \text{ there exists } \underline{x} \in S^n \text{ such that}$$

$$\xi(0_i; \underline{x}) < \xi(m_i; \underline{x}).$$

We thus have the following characterization.

THEOREM 3.6 - The function $\xi: S^n \rightarrow S$ is a BW multistate system structure iff (3.1), (3.3), (3.5) and (3.6) hold.

PROOF - Necessity is again obvious. To prove sufficiency all we have to show, in virtue of Theorem 3.5, is that

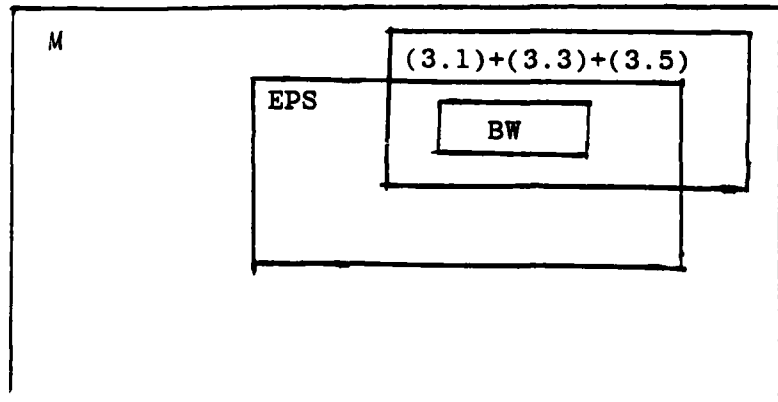
$$U\{P_1(\underline{w}): \underline{w} \in P_1\} = C.$$

Fixing $i \in C$ it follows from (3.6) that there exist $x \in S^n$ such that

$$\xi(0_i, \underline{x}) < k \leq \xi(m_i, \underline{x})$$

for some k , $k=1,2,\dots,m$. From Theorem 3.2 there exist $\underline{z} \in P_k$ such that $\underline{z} \leq (m_i, \underline{x})$ and obviously $z_i > 0$, since otherwise $\underline{z} \leq (0_i, \underline{x})$ and $\xi(\underline{z}) < k$ (recall that $\underline{z} \in P_k \Rightarrow \underline{z} \in \{0, k\}^n$). Therefore $i \in P_k(\underline{z})$ or equivalently $i \in P_1(\frac{1}{k}\underline{z})$ by Lemma 3.4, and the proof is complete.///

In the diagram below we depict the various classes of multi-state system structures involved in our discussion.



4. FINAL REMARKS

1) From the observation made right after Theorem 3.5, it follows that the class functions satisfying condition (3.1), (3.3) and (3.5) includes functions $\xi: S^n \rightarrow S$ which may be constant in some of its arguments. In other words, it includes multistate system structures with "inessential" components. Condition (3.6) enters here to require that every component be essential in some sense. This condition was first used by Griffith (1979) to define weakly coherent multistate system structures. Under property P it follows from this additional condition that $\bigcup_{\underline{y} \in P_1} P_1(\underline{y}) = C$.

A stronger result can actually be stated:

Proposition 4.1. Let $\xi: S^n \rightarrow S$ is an MSS and $k \in S - \{0\}$. Then

$$U\{P_k(\underline{w}) : \underline{w} \in P_k\} = C$$

if and only if for every $i \in C$ there exists $x \in S^n$ such that

$$\xi(0_i, \underline{x}) < k \leq \xi(m_i, \underline{x}). \quad ///$$

The above result suggests a new notion of component relevance which can be stated as

(4.1) "For every $i \in C$ and $k \in S - \{0\}$ there exists $\underline{x} \in S^n$ such that $\xi(0_i, \underline{x}) < k \leq \xi(m_i, \underline{x})$ ".

This requires, in some sense, that every component be relevant for system performance at all levels.

Recall that in (3.6) we introduced a notion of component relevance due to Griffith (1981) in order to characterize a BW MSS; which is weaker than the one above. However in the presence of property P, we have

Proposition 4.2. Under property P, the notions of component relevance (3.6) and (4.1) are equivalent. ///

2) It is interesting to remark that the class of EPS multistate system structures for which property P hold does reduce to the BW ones in the simple case where $m = n = 2$. However, as the following example shows, this is not true in general.

EXAMPLE - $S = \{0, 1, 2\}$, $C = \{1, 2, 3\}$

\tilde{x}	$\xi(\tilde{x})$
(2,2,2)	2
(2,1,2)	2
(1,2,2)	2
(2,2,1)	1
*(2,2,0)	1
(2,0,2)	2
(0,2,2)	2
(2,1,1)	1
(1,2,1)	1
(1,1,2)	1
(2,0,0)	0
*(0,2,0)	1
(0,0,2)	0
(2,1,0)	1
(2,0,1)	1
(1,0,2)	1
(1,2,0)	1
(0,2,1)	1
(0,1,2)	1
(1,1,1)	1
(1,0,1)	1
(1,1,0)	1
(0,1,1)	1
(1,0,0)	0
(0,1,0)	1
(0,0,1)	0
(0,0,0)	0

This function ξ verifies the conditions of definition 2.1 and therefore is an EPS multistate system structure. It can also be checked that property P holds for the function ξ . However the two starred points show that (3.5) is not true and that ξ is not of the B-W type.

3. After this paper had been written, it came to our knowledge that other forms of characterizing BW multistate system structures were developed by Natvig (1981) and Block & Savits (1981). Their results however differ from ours in the sense that no explicit directly verifiable conditions on ξ are given. We add however the following additional remarks that relate our results with the Natvig characterization.

Assume that $\xi: S^n \rightarrow S$ is an MSS and that for some $k \in S - \{0\}$ the following property is verified:

(4.2) "If $\xi(\underline{x}) \geq k$, there exists $\underline{z} \in \{0, k\}^n$ such that $\underline{z} \leq \underline{x}$ and $\xi(\underline{x}) \geq k$ ", i.e. property P introduced in the preceding section holds just for some level $k \in S - \{0\}$. It is easy to see that if $k \in S - \{0\}$ is one of the levels for which (4.2) holds and $\underline{x} \in P_k$, we must have $\underline{x} \in \{0, k\}^n$ and consequently

$$\xi(\underline{x}) \geq k \text{ iff } \max_{\underline{w} \in P_k} \min_{i \in P_k(\underline{w})} x_i \geq k,$$

which again follows from Theorem 3.2. This can be reworded as

$$\xi(\underline{x}) \geq k \text{ iff } \xi_k(\alpha_k(\underline{x})) = 1$$

for some binary, not necessarily coherent, monotone structure

function ξ_k , where $\alpha_k(\underline{x}) \in \{0,1\}^n$ has i -th component equal to 1 iff $x_i \geq k$. Furthermore, it follows from proposition 4.1 that the binary monotone structure function ξ_k will be coherent iff for every $i \in C$ there exist $\underline{x} \in S^n$ such that

$$\xi(0_i, \underline{x}) < k \leq \xi(m_i, \underline{x})$$

We recall that an MSS $\xi: S^n \rightarrow S$ is defined by Natvig to be a multistate coherent system of type 2 iff there exist binary coherent monotone structure functions $\xi_1, \xi_2, \dots, \xi_m$ such that

$$\xi(\underline{x}) \geq j \iff \xi_k(\alpha_k(\underline{x})) = 1, \quad k = 1, 2, \dots, m.$$

From the observation made above we have

THEOREM 4.3 - An MSS $\xi: S^n \rightarrow S$ is a multistate coherent system of type 2 iff (3.3) and (3.6) hold.

PROOF - Follows immediately from above observations and proposition 4.2. ///

As a final remark we add the following more explicit result that combines our approach with that of Natvig's, whose proof we omit.

THEOREM 4.4 - A multistate coherent system of type 2, $\xi: S^n \rightarrow S$ is a BW-MSS iff $\xi(\{0,m\}^n) = \{0,m\}$.

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